Operational risk measurement beyond the loss distribution approach: an exposure-based methodology

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Abstract

The Loss Distribution Approach (LDA) has evolved as industry standard for operational risk (OR) models despite a number of known weaknesses. In particular, LDA's traditional focus on historical loss data often neglects expert knowledge that is available for OR types of a more predictable nature. In this paper, we present an alternative quantification technique, so-called exposure-based operational risk (EBOR) models, which aim at replacing historic severity curves by measures of current exposures as well as using event frequencies based on actual exposures instead of historic loss counts. We introduce a general mathematical framework for exposure-based modeling that is applicable to a large number of operational risk types. As an example, an EBOR model for litigation risk is presented. Furthermore, we discuss the integration of EBOR and LDA models into hybrid frameworks facilitating the migration of OR subtypes from a classical to an exposure-based treatment.

The implementation of EBOR models is a challenging task since new types of data and a higher degree of expert involvement are required. In return, EBOR models provide a transparent quantitative framework for combining forward-looking expert assessments, point-in-time data (e.g. current portfolios) and historical loss experience. Individual loss events can be modeled in a granular way, which facilitates the reflection of loss-generating mechanisms and provides more reliable signals to risk management.

Keywords: Operational Risk, Loss Distribution Approach, Exposure, Factor Model, Litigation Risk

Contents

1 Introduction

The Basel Committee, cf. (Basel Committee on Banking Supervision, 2004), defines operational risk (OR) "as the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events." The definition "includes legal risk, but excludes strategic and reputational risk."

By its very nature, the operational risk of a typical bank is dominated by low-frequency high-severity events, i.e. single extreme losses (McNeil et al., 2005). This feature poses a key challenge to OR modeling since it requires the accurate reflection of heavy tails of loss distributions. Two main approaches have emerged in the industry to tackle this problem. The scenario-based approach mainly relies on the knowledge of subject-matter experts who specify loss scenarios for the relevant unit of measure, e.g. a cell within a business-line/event-type matrix. Due to the inherent subjectivity of human judgement this approach leads to high model uncertainty, in particular if scenarios are constructed for extreme and unlikely loss events. In many financial institutions, historic loss data has therefore been considered the most reliable source for modeling OR tail distributions. This methodology, called the Loss Distribution Approach (LDA), has evolved as an industry standard for OR models satisfying the requirements of the Advanced Measurement Approach (AMA) under Basel II regulations, cf. (Basel Committee on Banking Supervision, 2009).

The fundamental premise behind LDA is that each firm's operational losses are a reflection of its underlying operational risk exposure. In fact, for those operational risks for which historical loss distributions are assumed to be the best predictor of future losses, LDA models perform reasonably well. However, strong reliance on the historical loss experience has a number of serious disadvantages. Models based on historic loss data have an inherently backward-looking character and are not well linked to the loss-generating process and control environment. In the wider AMA framework, business environment and internal control factors (BEICF) are required by Basel II regulations, however, this element is usually still insufficient to fully capture exposures or forward-looking aspects. As a consequence, capital estimates are too slow in adapting to changes in the risk profile, e.g. due to the introduction of new products or changes in the business mix of the bank (e.g. divestments), and do not provide sufficient incentives for OR management to mitigate risk.

The industry has also observed LDA model instability, and thus capital volatility, due to non-robust model parameters with respect to the inclusion of data or methodology changes necessitated by the data. No standards for consistent usage of input data or LDA model specification have emerged in the past, which makes it difficult to compare LDA-based capital estimates calculated at different financial institutions.

The high uncertainty in OR capital estimates has triggered a regulatory debate about replacing the Advanced Measurement Approach by a rather simplistic Standardized Measurement Approach (SMA) in a future regulatory regime, see

(Basel Committee on Banking Supervision, 2016). Not surprisingly, recent studies have shown that the SMA suffers from a number of deficiencies, which make it unsuitable for a realistic quantification of operational risks and limit its applicability in OR management, see (Cope et al., 2016), (Chapelle et al., 2016).

Given the weaknesses of the current OR model landscape, it is a main priority for OR management to investigate alternative modeling techniques. The objective is not only to overcome some of the deficiencies for regulatory capital calculation listed above but to develop models that satisfy emerging requirements of risk measurement such as the annual Comprehensive Capital Analysis and Review (CCAR) performed by the FED, see (Board of Governors of the Federal Reserve System, 2012), and the EU-Wide Stress Tests, see (European Banking Authority, 2011).

A promising approach in this respect are so-called exposure-based operational risk (EBOR) models which aim at replacing unbounded historic severity curves by measures of current exposures (*maximum possible loss*) as well as using event frequencies based on actual exposures (*maximum number of events*) instead of historic loss counts. The objective of this paper is to introduce a general mathematical framework for exposure-based modeling that is applicable to a large number of operational risk types. As our prime example, we use an EBOR model to quantify operational risk for a portfolio of pending litigations. In many ways, this risk is particularly well suited for an exposure-based treatment within the OR framework. First of all, the pending litigations clearly specify the potential loss events that have to be captured in the model. Secondly, for each case there exists an estimate for the *exposure* and subject matter experts (SME) in the bank typically have an educated view on the probability and range of outflow, which can then be translated into the model parameters *event probability* and the stochastic *loss-given-event* ratio. Techniques from credit portfolio modeling are used to specify the loss distribution of the portfolio of pending litigations taking dependencies between individual litigations into account.

The development, calibration and validation of EBOR models typically is a challenging task since new types of data and a higher degree of expert involvement across the institution are required. In return, EBOR models provide a transparent quantitative framework for combining forward-looking assessments of subject-matter experts, point-in-time data (e.g. current portfolios) and historical loss experience. Individual events can be modeled in a more granular and comprehensive way than in LDA models, which facilitates a better reflection of loss-generating mechanisms as well as risk mitigants. The increased model granularity combined with forward-looking expert assessment leads to a more realistic dynamic of capital estimates providing more reliable signals to risk management and fostering the dialogue with risk management and business experts.

OR quantification techniques using exposure- or factor-based concepts are currently investigated in the finance industry, exemplifying modeling for rogue trading, sales practices and other risks. Capital demand, stress testing (including CCAR) and risk appetite are highlighted as (potential) applications, see (Baruh, 2016). We also refer to (Yan and Wood, 2017), a recent publication on a structural model associated with the mis-selling of retail banking products.

This paper starts with a short review of LDA models in section 2. We then introduce the general concept behind EBOR models and provide arguments why the exposure-based models are a promising approach to remediate some of the LDA shortcomings. The different components of an EBOR model are presented in more detail in section 3. Section 4 deals with the application of EBOR techniques to a portfolio of pending litigations. The integration of EBOR models into an LDA framework is analyzed in section 5. Section 6 concludes.

2 An Exposure-Based Approach to Operational Risk Modeling

2.1 Shortcomings of the Loss Distribution Approach

The starting point of our EBOR development is a short review of LDA models, which is motivated by the fact that the Loss Distribution Approach, as industry standard, is a natural benchmark for new models. Secondly, a complete replacement of LDA models by exposure-based techniques is neither realistic short-term and may not be feasible at all for some risks. It is therefore important to develop techniques for a meaningful integration of both concepts (cf. section 5).

The Loss Distribution Approach is based on actuarial techniques, see (McNeil et al., 2005) for a comparison of different modeling approaches. The basic idea is to partition OR loss data into sufficiently homogeneous sets, typically corresponding to combinations of n OR event types (ET) and business lines (BL), and to calibrate a frequency and a severity distribution for each BL/ET combination. These distributions specify the loss variable X_j for the j-th BL/ET combination through a compound sum, i.e.,

$$X_j = \sum_{k=1}^{N_j} S_{jk}, \qquad X = \sum_{j=1}^{n} X_j, \tag{1}$$

where the frequency variable N_j and the severity variables $S_{j1}, S_{j2}, \ldots$ follow the respective frequency and severity distributions. The loss variable X on bank level is then obtained by aggregating the X_j based on the dependence structure of the underlying variables, see (Aue and Kalkbrener, 2006) for a comprehensive presentation of a LDA model implemented in a bank.

As stated in an industry position paper, see (The AMA Group, 2013), the LDA provides a rigorous approach for modeling past loss distributions. It has become the standard practice for modeling those operational risks for which historical loss distributions are assumed to be the best predictor of future losses. For other risk types that have more predictable characteristics, at least for the next twelve months, it is problematical to rely almost exclusively on the historical loss experience. The AMA Group provides a spot-on analysis of the fundamental concerns related to LDA models, if applied to 'predictable' risk types such as litigations:

Certain operational risk types have emerged as quite material over recent years and have proven to be problematical for LDA from a representation-of-risk standpoint.

Litigation events linked to credit or market risk losses emerged during the recent crisis as material sources of operational risk. Many of these events are related to representations and warranties on sold mortgages that defaulted during the crisis. The risk exposure for these events is defined by the credit or market risk exposure in contrast to standard operational risk types with an undefined exposure, and although operational in nature the losses can be driven by credit events such as defaults. Additionally, predictive factors for the operational risks are not captured in LDA, but could be assigned using a combination of statistical modeling and expert judgment, allowing for factor based quantification of capital requirements.

The use of LDA for these 'predictable' risk types has been observed to undercapitalize known risks before they occur, and overcapitalize for risk after the losses materialize, creating inappropriate capital estimates, including:

- *Underestimation at the time of manifestation of loss due to lag in 'realizing' losses for events that are already known to have been triggered. This is particularly relevant for large litigation losses for which there is a significant lag between the event trigger and the initial reserve.*
- *Overestimation of capital estimates in a time lag after the manifestation of loss due to extrapolation to the 99.9th percentile and an over-stretched distribution.*
- *The large data gap is a modeling issue/challenge and capital results are questionable.*

An unintended consequence of this timing paradox is that it results in disincentives to taking strong risk management steps to mitigate risk. Where LDA models drive capital and risk management, capital will at times increase in tandem with risk mitigation steps, a counterintuitive phenomenon, thus being at odds with strong risk management.

The industry has observed LDA model instability, and thus capital volatility, due to the non-robust model parameters relative to inclusion of the data or in combination with a methodology change of some type necessitated by the data (e.g. distribution change, fitting routine change). The implications can be:

- *A disconnect with Market and Credit Risk practices, due to not using generally accepted risk measurement techniques.*
- *Loss of credibility with Senior Management due to the lack of transparency and inability to apply their intuition to understand the model results.*

- *Reaction to public disclosure (Pillar 3) – industry and regulators alike must be sensitive to analyst review and reporting. Important for industry, of course, to deal with undesired market effects on stakeholders, and also important for regulators to deal with undesired consequences.*

In addition to the shortcomings listed above, LDA models are typically not well suited for stress testing or loss projection under macroeconomic or idiosyncratic scenarios as required by EBA stress tests or the FED's Comprehensive Capital Analysis and Review (CCAR). The main obstacle for implementing stress scenarios is the missing (direct) link between macro or market drivers and the components of the LDA model, e.g. frequency and severity distributions. The application of LDA models in legal entity risk management is an even more challenging task due to increasing data scarcity issues.

2.2 Exposure-based OR modeling

The concerns raised in the previous section motivate the development of exposure-based modeling techniques to complement and (where possible) substitute existing LDA models.

In the following, a formal presentation of the basic structure of those EBOR models will be provided. We consider n potential loss events, where n can be considered as frequency exposure since it specifies the maximum number of loss events. Their interpretation depends on the scope of the specific EBOR model, i.e. on the OR subtype to be covered. For each of these events there exists a Bernoulli variable I_j, $j = 1, \dots, n$, such that $\{I_j = 1\}$ represents the occurrence of the j-th event in the period $[0, T]$, where T is typically set to one year. The probability $p_j := \mathbb{P}(I_j = 1)$ is the corresponding event (or loss) probability. We assume that for each loss event a maximum positive loss amount E_j can be specified, which is called the severity exposure of the j-th event. Since only a fraction of the exposure is typically lost, a random variable L_j is specified, which specifies the loss ratio or loss-given-event (LGE) as percentage of exposure.

The aggregate event loss variable of the exposure-based operational risk model is now given by

$$Y := \sum_{j=1}^{n} Y_j \tag{2}$$

with individual losses

$$Y_j := I_j \cdot L_j \cdot E_j, \quad j \in \{1, \dots, n\}. \tag{3}$$

In order to reflect dependencies between the occurrence of different loss events, the event indicators I_j are typically modeled as dependent variables. A frequently used approach is the specification of dependencies between events through the introduction of risk factors. This concept is formalized in the definition of a Bernoulli mixture model, see (McNeil et al., 2005).

Given a vector of real-valued random variables $\Psi = (\Psi_1, \dots, \Psi_m)$, the risk factors, the random vector $(I_1, \dots, I_n)$ follows a Bernoulli mixture model with factor vector Ψ, if $m < n$ and there exist functions $\mathrm{cp}_j : \mathbb{R}^m \to [0,1]$, $1 \le j \le n$, such that conditional on Ψ the random vector $(I_1, \dots, I_n)$ is a vector of independent Bernoulli random variables satisfying $\mathbb{P}(I_j = 1 \mid \Psi = \psi) = \mathrm{cp}_j(\psi)$ for $\psi \in \mathbb{R}^m$. Hence, for any $y = (y_1, \dots, y_n) \in \{0,1\}^n$ and $\psi \in \mathbb{R}^m$, the conditional event probabilities are given by

$$\mathbb{P}\left(I_1 = y_1, \dots, I_n = y_n \mid \Psi = \psi\right) = \prod_{j=1}^{n} \mathrm{cp}_j(\psi)^{y_j} \left(1 - \mathrm{cp}_j(\psi)\right)^{1-y_j}. \tag{4}$$

Further risk factors $\Psi_{m+1}, \dots, \Psi_l$ might be required to specify dependencies between the loss ratios $L_1, \dots, L_n$. The precise functional form between the Ψ_i and the L_j depends on the specific application of the EBOR model.

Risk factors not only introduce dependencies between the basic variables I_j and L_j of an EBOR model. As long as the risk factors have an (economic) interpretation these factors can be used to implement stress scenarios in EBOR models. More precisely, if a stress scenario is specified for some of the factors $\Psi_1, \dots, \Psi_l$ then its impact can be quantified by performing an EBOR calculation conditional on the stressed values or stressed distributions of Ψ.

Note that the formal EBOR definition is similar to credit portfolio models based on default thresholds. In these models, e.g. CreditMetrics, cf. (Bhatia et al., 1997), and Moody's KMV Portfolio Manager, cf. (Bohn and Crosbie, 2002), the portfolio loss of a credit portfolio due to defaults is also specified by the loss variables (2) and (3), only the economic interpretation of the variables is adjusted to a credit risk setting: I_j now denotes the default indicator of the j-th counterparty, E_j is the (deterministic) credit exposure and L_j is the (stochastic) loss-given-default. Analogously to (4), joint default probabilities are specified through a Bernoulli mixture model. The systematic factors $\Psi_1, \dots, \Psi_m$ frequently represent countries (or geographic regions) and industries and are chosen to reflect credit concentrations in the underlying portfolio.

2.2.1 EBOR application scope and examples

The general EBOR concept and its relation to frequencies I_j and severity ratios L_j as well as risk factors ψ, the frequency exposure n and severity exposures E_j is depicted in Figure 1.

The model framework can be applied to rather different OR sub-risks: from very inhomogeneous portfolios of infrastructure or litigation risks with specific character-

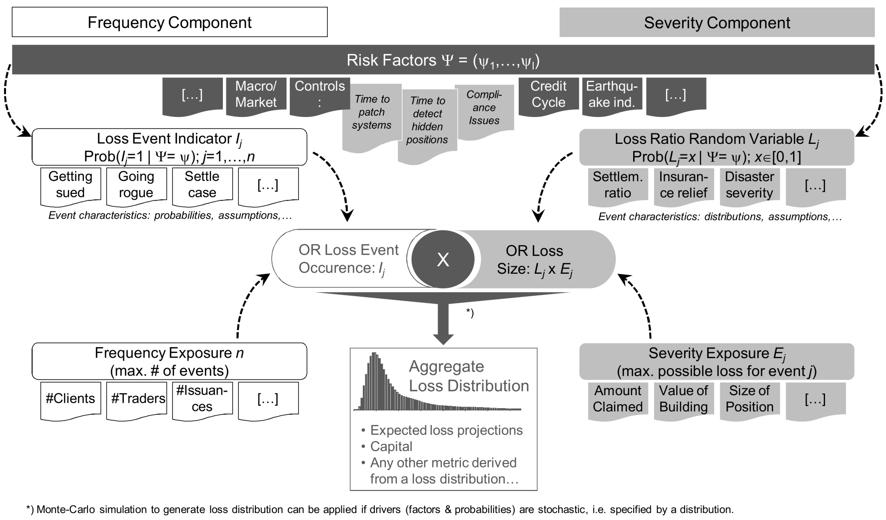

Figure 1: General framework for EBOR models highlighting the frequency, severity and risk factor components. For each quantity some examples are given in the chart.

istics of individual events to portfolios of rogue trading risks with common characteristics for the events to be simulated. Even for situations with potentially very large – but not infinite – exposures an EBOR model can be used. A few examples are listed below.

Natural disaster: EBOR models for natural disaster impact on buildings could be based on *number of buildings* as frequency exposure and *value of buildings* as severity exposure using building-specific characteristics such as earthquake protection, region-specific earthquake indicators and typical earthquake severities as risk factors.

Rogue trading: A rogue trading EBOR model could take a specific group of traders as frequency exposure with a homogeneous probability of *going rogue* into account and then model for each rogue trading event the severity based on *size of the hidden position* as severity exposure and *time to detection* or *market movement* as severity risk factors.

Litigation: In section 4 we employ the general EBOR framework to develop a model for pending litigations, i.e. the event triggering the filing of the litigation has already happened and only the final outcome of the court case has to be modeled. Conceptually, this model can be extended to also include potential future litigations, e.g. based on credit properties of an underlying issuance portfolio, c.f. (Rosa, 2012).

2.3 EBOR modeling – challenges and rewards

From a formal point of view, the basic EBOR model introduced in the previous subsection can be considered as a special case of the LDA model specified in (1): the frequency variables N_j have to attain values in $\{0, 1\}$ and the severity variables have to be bounded. In this special case, the n different components of the LDA model no longer correspond to BL/ET combinations but to individual loss events. Their event probabilities are defined by $\mathbb{P}(N_j = 1)$.

Despite these formal similarities, the calibration and application of LDA and EBOR models are different in many aspects. In general, the development, calibration and validation of EBOR models is a challenging task since new types of data and a higher degree of expert involvement across the institution are required. In return, EBOR models provide a transparent quantitative framework for combining forward-looking assessments of subject matter experts, historical loss experience and point-in-time data (e.g. current portfolios) instead of relying mainly on historic loss data. As a consequence, EBOR models have a number of advantages that resolve many of the issues listed in section 2.1. Individual events can be modeled in a more granular and comprehensive way than in LDA models, which facilitates a better reflection of loss-generating mechanisms as well as risk mitigants. The increased model granularity combined with forward-looking expert assessment leads to a more realistic dynamic of capital estimates, i.e. EBOR modeling typically reduces the problem of undercapitalization of known risks at an early stage and the subsequent overcapitalization after the losses materialize, see section 4.4 for a specific example. This feature incentivises risk management to take appropriate risk mitigating actions.

The transparency of EBOR models extends their scope beyond the quantification of risk capital, e.g. supporting risk-based decision processes by the capability to price individual transactions or expansion into new market segments. Furthermore, EBOR models facilitate communication between quants, risk managers and business experts (including legal and compliance departments etc.) and ensure that discussions aim at identifying the underlying risk drivers (and potential management of those drivers) instead of debating solely about historic losses. In our experience, non-quant experts are more willing to share expertise and data from their day-to-day business as input into EBOR models than to accept statistical relationships under LDA if they have difficulties to understand the link to the actual (perceived) risk exposure (*implausible model sensitivities*).

The development of EBOR models requires specific knowledge about potential loss events and the underlying loss-generating mechanisms. Expert knowledge plays a central role for the parametrization of the model, in particular because granular information is required for modeling individual events, e.g. event probabilities and exposures. Historical data is mainly used to supplement and validate/verify expert assessment.

A sensible route for a (partial) transition from LDA to EBOR could be based on the following steps:

1. *Identification and Classification:* Identify risk types (OR taxonomy) and order by materiality

2. *Model Development:* Build EBOR models step-by-step for those risks where sufficient information is available (starting with material risks)

3. *Completeness Check:* Use LDA for remaining risks

4. *Aggregation:* Perform a meaningful aggregation of risks across EBOR and LDA models

Even if carefully designed EBOR models are applied, capital numbers at extreme quantiles still suffer from a high level of uncertainty. The models themselves, however, are more transparent and more closely related to loss-generating mechanisms. They also show plausible sensitivities. As a consequence, EBOR models facilitate a better alignment to risk management and extend the scope of applicability beyond the quantification of risk capital.

3 EBOR model details

The basic concept of an EBOR model has been introduced in the previous section, see definitions (2) and (3). The main parameters of the model are the exposure E_j, the loss ratio L_j and the event indicator I_j whose product equals the loss variable Y_j. One objective of this section is to discuss in more detail how to model these parameters. In particular, we propose techniques for reflecting dependencies between event indicators as well as loss ratios, see sections 3.1.1 and 3.1.2.

Not all losses in an EBOR model are necessarily triggered by event indicators, e.g. legal fees might be independent of the outcome of a litigation. In order to capture this aspect, we present a model extension in section 3.2 that specifies "non-event" losses.

Since techniques for risk mitigation like third party insurance or indemnification play a major role in operational risk management we investigate the implementation of risk mitigants in EBOR models in section 3.3. Deduction of existing provisions to reduce risk of future payments is also possible under EBOR and described in section 3.4. Furthermore, we provide information on the calculation of risk measures and capital allocation in EBOR models. The related section 3.5 additionally contains a detailed step-by-step procedure for the simulation of the aggregate loss distribution under an EBOR model.

3.1 EBOR event losses

3.1.1 Modelling correlated EBOR events

In order to reflect dependencies between the occurrence of different loss events the event indicators I_j are typically modeled as dependent variables. A powerful dependence concept are Bernoulli mixture models, see section 2.2. For its specification, risk factors $\Psi_1, \dots, \Psi_m$ have to be identified that introduce dependencies between the event indicators $I_1, \dots, I_n$ with individual event probabilities $p_1, \dots, p_n$.

Subject matter experts typically play an important role in specifying dependencies, in particular since historical data is often too scarce to allow for a purely statistical approach. In order to facilitate the use of expert knowledge we apply a degenerate version of a Bernoulli mixture model such that, conditionally on $\Psi = \psi$, the event indicators $I_1, \dots, I_n$ become deterministic. The definition of the risk factors is based on partitioning events into independent clusters, i.e. the factor Ψ_j represents the state of the j-th cluster and factors $\Psi_1, \dots, \Psi_m$ are independent. Typically, this kind of information can be more easily provided by subject matter experts. The clustering approach leads to a particularly simple distribution function for the event frequency variable $\sum_{j=1}^{n} I_j$, which also facilitates the integration of an EBOR into an LDA model, see section 5.

More precisely, we assume that the n loss events are split into clusters $C_1, \dots, C_m$. Formally, $C_1, \dots, C_m$ is a partition of $\{1, \dots, n\}$, i.e.

$$\bigcup_{i=1,\dots,m} C_i = \{1, \dots, n\}, \;\; C_i \cap C_j = \emptyset \;\text{ if }\; \mathrm{i} \neq \mathrm{j}.$$

We make two assumptions:

Maximum dependence within a cluster: For each cluster C_i, $1 \leq i \leq m$, a continuous random variable $\Psi_i : \mathbb{R} \to [0, 1]$ with distribution function F_{Ψ_i} is specified. We require that for each $j \in C_i$ the conditional probabilities cp_j can be written as

$$\mathrm{cp}_j(\psi_1, \dots, \psi_m) = \begin{cases} 0 & , \quad \psi_i > F_{\Psi_i}^{-1}(p_j) \\ 1 & , \quad \psi_i \leq F_{\Psi_i}^{-1}(p_j) \end{cases}$$

Independence of clusters: We assume that $\Psi_1, \dots, \Psi_m$ are independent of each other. This assumption can be easily relaxed if required. It mainly reflects that clusters of events, e.g. litigations on unrelated issues or natural disasters in different regions, are typically uncorrelated. The assumption also leads to a simple closed form of the total event distribution function, see equation (9) below, and therefore to a smooth integration of EBOR modelling into an LDA model.

Due to the first assumption, the occurrence of the j-th loss event only depends on the random variable Ψ_i of the embedding cluster C_i, $j \in C_i$, and the event-specific threshold $F_{\Psi_i}^{-1}(p_j)$: $I_j = 1$ if and only if $\Psi_i \leq F_{\Psi_i}^{-1}(p_j)$ is fulfilled. Hence, the randomness of events is fully captured by the random variables $\Psi_1, \dots, \Psi_m$ and the conditional event probabilities in (4) are either 0 or 1.

In the following, we will formalize the consequences of our model assumptions for the likelihood of joint events. For any pair $j_1 \in C_{i_1}$ and $j_2 \in C_{i_2}$ the joint event probability is

$$\mathbb{P}\left(I_{j_1} = 1, I_{j_2} = 1\right) = \begin{cases} \min\{p_{j_1}, p_{j_2}\} & , \quad i_1 = i_2 \\ p_{j_1}\, p_{j_2} & , \quad i_1 \neq i_2 \end{cases} \tag{5}$$

and, hence, the implicit (linear) event correlation equals

$$\mathrm{Corr}(I_{j_1}, I_{j_2}) = \begin{cases} \sqrt{\frac{p_{j_1}\ (1-p_{j_2})}{p_{j_2}\ (1-p_{j_1})}} & , \quad i_1 = i_2 \\ 0 & , \quad i_1 \neq i_2 \end{cases} \tag{6}$$

where we have assumed, w.l.o.g., $p_{j_1} \leq p_{j_2}$. This implies that loss events in the same cluster, which have identical event probabilities, are perfectly correlated. If their event probabilities are different, the occurrence of these events still has the maximum degree of correlation that can be achieved. Events in different clusters are uncorrelated.

Next we derive the distribution function for the event frequency variable $I_P := \sum_{j=1}^{n} I_j$. For each cluster $i \in \{1, \dots, m\}$, we define the corresponding event frequency variable and its (discrete) density function:

$$I_{C_i} := \sum_{j \in C_i} I_j, \quad f_i(k) := \mathbb{P}(I_{C_i} = k),\ k = 0, \dots, n.$$

For each $i \in \{1, \dots, m\}$, we order the elements $j_1, \dots, j_{|C_i|}$ of cluster C_i with decreasing event probabilities

$$p_{j_1} \geq p_{j_2} \geq \dots \geq p_{j_{|C_i|}}$$

and obtain from the assumption on maximal dependence within a cluster

$$f_i(k) = \begin{cases} 1 - p_{j_1} & , \quad k = 0 \\ p_{j_k} - p_{j_{k+1}} & , \quad k = 1, \dots, |C_i| - 1 \\ p_{j_k} & , \quad k = |C_i|. \end{cases} \tag{7}$$

The cumulative density function g_i covering the first i clusters is defined by

$$g_i(k) := \mathbb{P}(\sum_{j=1}^{i} I_{C_j} = k),\ k = 0, \dots, n.$$

Note that g_m is the density of I_P, i.e. $g_m(k) = \mathbb{P}(I_P = k)$.

There exists a simple recursion formula for the cumulative density g_i. By definition, $g_1 = f_1$. Since clusters are supposed to be independent, we have

$$g_i(k) = \sum_{j=0}^{k} g_{i-1}(j)\, f_i(k-j) \tag{8}$$

$$= \sum_{(k_1,\dots,k_i)\in K(i,k)} \prod_{j=1}^{i} f_j(k_j), \tag{9}$$

where

$$K(i,k) := \{(k_1,\dots,k_i) \in \{0,1,\dots,n\}^i \mid \sum_{j=1}^{i} k_j = k\}.$$

Recursion (8) will be used for a joint LDA and EBOR simulation, cf. section 5.2.

3.1.2 Modelling correlated EBOR losses

Conditional on the occurrence of an event, i.e. $I_j = 1$, we determine the amount of outflow as the product of exposure E_j and loss-given-event ratio L_j. While the exposure is deterministic and represents the maximal loss, the variables $(L_1,\dots,L_n)$ follow a multi-variate distribution with individual marginal distributions, e.g. specified by expected value and volatility and an overall dependence structure.

EBOR models are rather flexible in terms of specifying the dependence structure of the loss-given-event ratios $L_1,\dots,L_n$. Analogously to dependent events, risk factors as well as copula functions can be used for this task, see (McNeil et al., 2005). If dependencies are specified through risk factors with an economic interpretation the behaviour of LGE ratios can be potentially linked to event indicators. It depends on the specific application of the EBOR model whether an identification of adequate risk factors is feasible.

For modelling dependent events we have proposed a simple clustering approach, cf. section 3.1.1. Assuming that loss-given-event ratios within a cluster show a rather homogeneous behaviour these clusters could also be utilized for specifying a dependence structure for LGE ratios. One option is to estimate intra- and inter-cluster correlations and to define either risk factors or copulas to implement the correlation structure in the model.

As for their dependence structure, EBOR models do not restrict the choice of (marginal) distribution functions for LGE ratios. In the literature, (truncated) beta or log-normal distributions are frequently used, see (Memmel et al., 2012; Kaposty et al., 2017) for stochastic LGD models in the context of credit risk. Examples in the context of EBOR are provided in section 4.3.

3.2 EBOR non-event losses

For some risk types there are loss components that, although not conditional on the occurrence of an event, are indirectly linked to it. An important example are legal fees, which are an essential part of litigation risk. These fees are assumed to be paid on a regular basis regardless whether a litigation has been settled or not. To capture losses of this kind we extend the loss definitions of the EBOR model in the following way: for each portfolio constituent $j \in \{1, \dots, n\}$ we extend the loss variable defined in (3) by adding a second component that does not depend on the event indicator, i.e. we define the extended loss variable

$$Z_j^{(0)} = Y_j + Y_j^{(\mathrm{nE})}, \tag{10}$$

where $Y_j^{(\mathrm{nE})}$ specifies a non-event loss. More precisely, $Y_j^{(\mathrm{nE})}$ covers the aggregated amount of losses that are expected to occur within the relevant time horizon and can be considered closely related to – although not triggered by – the j-th loss event. For instance, Y_j and $Y_j^{(\mathrm{nE})}$ might have the same economic root cause or legal case, see section 4.3.2 for details.

The aggregate event loss variable Y of the exposure-based operational risk model is extended to

$$Z^{(0)} := \sum_{j=1}^{n} Z_j^{(0)} = \sum_{j=1}^{n} Y_j + Y_j^{(\mathrm{nE})}.$$

3.3 Risk mitigation

The standard risk mitigating tools for OR are designed to transfer the risk – or parts of it – to a third party. This could, for instance, be achieved through insuring certain OR losses or by indemnification of losses by liable third parties. Any such activity clearly has to account for the uncertainty of the third party to make its payment.

Under LDA the set of options is, in practice, limited to insurance contracts. The key constraint is that the model simulates a total number of events but does not provide specific information on their nature. Hence, it is, for instance, impossible to reflect the mitigating effect of event-specific indemnification (or, conversely, there is a risk that the indemnitor cannot fulfill its liabilities). In contrast, the standard LDA provides the required granularity for reflecting insurance contracts, see (Aue and Kalkbrener, 2006), since insurance categories can be mapped to OR loss categories in an LDA model, e.g. to OR event types, business lines or a combination of both. If the losses of one operational risk category are covered by several insurance policies, the percentage of losses that fall into a specific insurance category need to be determined as well. For example, 70% of execution losses might be covered by General Liability, 20% by Professional Liability and 10% are not insured.

Under EBOR individual loss events are modeled. It is not necessary to specify coverage ratios for specific OR loss categories in insurance modelling and event-specific indemnification can be incorporated. Usually, risk mitigants are applied in the following order: The loss amount before risk mitigation, denoted by $Z_j^{(0)} = Y_j + Y_j^{(\mathrm{nE})}$ for $j \in \{1, \dots, n\}$, is adjusted to $Z_j^{(\mathrm{ind})}$ by applying case-specific indemnification of losses. Subsequently, $Z_j^{(\mathrm{ind})}$ is further reduced to $Z_j^{(\mathrm{ins})}$ if insurance exists for the j-th loss event. Note that loss indemnification and insurance are assumed to cover all expenses including non-event losses.

We finally point out that the EBOR model – unlike the LDA – allows for the deduction of existing provisions from loss $Z_j^{(\mathrm{ins})}$. The related methodology will be described in section 3.4.

3.3.1 Third party indemnification

We formalize the (partial) indemnification of event losses by a third party, a so-called indemnitor. Formally, the indemnitors $\mathrm{Ind}_1, \dots, \mathrm{Ind}_l$ form a partition of $\{1, \dots, n\}$ such that all events $j \in \mathrm{Ind}_k$ are covered by the k-th indemnitor. The default probability of the k-th indemnitor is denoted by $p_k^{(\mathrm{ind})}$ and $I_k^{(\mathrm{ind})}$ is the corresponding Bernoulli variable defined by $p_k^{(\mathrm{ind})} := \mathbb{P}(I_k^{(\mathrm{ind})} = 0)$. Finally, $\omega_j^{(\mathrm{ind})} \in [0, 1]$ is the indemnified proportion of the j-th event. It is set to zero if no indemnification is available. Hence, the j-th event loss after indemnification equals

$$Z_j^{(\mathrm{ind})} = \left(1 - I_k^{(\mathrm{ind})}\ \omega_j^{(\mathrm{ind})}\right)\ Z_j^{(0)},$$

where $j \in \mathrm{Ind}_k$. In other words, indemnification will be applied to all events $j \in \mathrm{Ind}_k$ if the k-indemnitor has not defaulted.

In the current model, we assume that the indemnification indicators $I_1^{(\mathrm{ind})}, \dots, I_l^{(\mathrm{ind})}$ are independent of each other and of the event losses $Z_1^{(0)}, \dots, Z_n^{(0)}$. This assumption could be relaxed by specifying a dependence structure for indemnitor defaults, e.g. by utilizing techniques from credit portfolio modeling.

3.3.2 Insurance

Since EBOR models reflect the specific characteristics of potential future loss events, event-specific contracts can be incorporated. In particular, it is not necessary to specify mappings between operational risk and insurance categories, which are required in LDA models due to their lower granularity, e.g. modelling BL/ET combinations instead of individual events.

Insurance contracts are characterized by certain specifications regarding the compensation of losses:

- A deductible d is defined to be the amount the bank has to cover by itself.

- The single limit l of an insurance policy determines the maximum amount of a single loss that is compensated by the insurer. In addition, there usually exists an aggregate limit l_{agg}.
- The methodology for recognizing insurance also has to take into account uncertainties related to insurance payments like the certainty and the speed of payment, insurance through captives or affiliates and the rating of the insurer. We translate the rating into a default probability $p^{(\text{ins})}$ and model defaults of the insurer by a Bernoulli variable defined by $p^{(\text{ins})} := \mathbb{P}(I^{(\text{ins})} = 0)$. The other limitations are transferred by expert judgement into a haircut H which is applied to the insurance payout.

In mathematical terms, the amount paid by the insurer is defined by

$$\text{Ins} := I^{(\text{ins})} \cdot \max(\min(l, y) - d, 0) \cdot H,$$

as a function of the loss amount y. The actual specification of insurance parameters for individual payouts Ins_j, $j = 1, \ldots, n$, including haircuts has to be based on insurance data and expert knowledge and is beyond the scope of this paper.

As for third party indemnification, we define a separate insurance indicator $I_k^{(\text{ins})}$ for each insurer and assume that different insurance indicators are independent of each other. In addition, the calculation of the payouts $\text{Ins}_1, \ldots, \text{Ins}_n$ has to reflect aggregate limits. This aspect is covered in the simulation process, see section 3.5.1.

Finally, the loss amount of the j-th event after indemnification and insurance is defined by

$$Z_j^{(\text{ins})} = Z_j^{(\text{ind})} - \text{Ins}_j.$$

3.4 Deduction of provisions

Provisions are used in accounting to pro-actively reflect expected payouts of certain events. The risk of significant future losses (in excess of the provision) is reduced. Unlike the LDA, the event-specific modelling of the EBOR approach allows for a better reflection of future cash flows (total losses minus provisions) making the forecasting more time-congruent.

More formally, we deduct the provision P_j for the j-th event from its loss-after-insurance $Z_j^{(\text{ins})}$ and arrive at the following loss definition

$$Z_j := Z_j^{(\text{ins})} - I_j \text{ P}_j.$$

We explicitly allow for gains if established provisions exceed the simulated loss.

3.5 Risk measures and capital allocation

One of the key benefits of the EBOR model is the possibility to determine risk contributions for each of the potential n loss events, i.e. portfolio constituents. This feature of the model is due to its granularity, i.e., individual loss events are modeled and therefore all the information is available that is required to allocate risk capital down to event level. The allocation of risk capital to loss events facilitates the identification of key risk drivers (or risk concentrations) and supports management actions to mitigate the risk.

The allocation of risk capital to individual events is derived from the risk capital calculated on aggregate level, which is based on the Monte Carlo simulation of the loss distribution of the EBOR model.

3.5.1 Simulation of the aggregate loss distribution

The aggregate loss distribution of an EBOR model cannot be represented in analytic form. We therefore apply Monte Carlo simulation to generate samples of the aggregate loss distribution Z, which are subsequently used for deriving risk measures on aggregate and event level:

$$Z = \sum_{j=1}^{n} Z_j = \sum_{j=1}^{n} \left(1 - I_k^{(\mathrm{ind})}\ \omega_j^{(\mathrm{ind})}\right)\ \left(\mathrm{LegalFee}_j + I_j \cdot L_j \cdot E_j\right) - \mathrm{Ins}_j - I_j\ \mathrm{P}_j.$$

These are the main steps in generating one MC sample, numbers in parentheses provide the link to Figure 2 depicting the simulation process flow.

Joint simulation of event indicators and loss ratios

(1) Generate a sample $(\psi_1, \dots, \psi_m)$ of the m-dimensional distribution of risk factors $\Psi_1, \dots, \Psi_m$ that specify the Bernoulli mixture model for the event indicators $I_1, \dots, I_n$.

(2) Conditionally on $(\psi_1, \dots, \psi_m)$, generate a sample $(i_1, \dots, i_n)$ of $I_1, \dots, I_n$.

(3) For those events that trigger losses, i.e. events specified by the index set $J := \{j \in \{1, \dots, n\} \mid i_j = 1\}$, simulate a sample of the joint distribution of loss ratios $(L_j)_{j \in J}$.

(4) Based on the simulated values of $I_1, \dots, I_n$ and $(L_j)_{j \in J}$, calculate the value of the loss vector $(Y_1, \dots, Y_n)$.

Simulation of non-event losses

(5) For each $j = 1, \dots, n$, add the non-event loss by simulating the corresponding variable $Y_j^{(\mathrm{nE})}$ resulting in a value of the loss variable $Y_j + Y_j^{(\mathrm{nE})}$ before risk mitigation denoted by $Z_j^{(0)}$.

Incorporation of risk mitigation and deduction of provisions

(6) Simulate indemnification and insurance indicators $I^{(\text{ind})}$ and $I^{(\text{ins})}$. Conditionally on the simulated values, calculate third party indemnification and insurance:

- For each $j = 1, \ldots, n$, compute the loss after indemnification $Z_j^{(\text{ind})}$.
- Apply the respective insurance contract to each event. Order all events randomly to take aggregate limits into account and calculate the loss after insurance $Z_j^{(\text{ins})}$ for all $j = 1, \ldots, n$.

(6) For all loss events $j \in J$, reduce $Z_j^{(\text{ins})}$ by the provision of the j-th loss event resulting in a sample value z_j of the loss variable Z_j after risk mitigation.

Sample of aggregate loss distribution

(7) The sum of the losses $z := \sum_{j=1}^{n} z_j$ is a sample of the aggregate loss distribution Z.

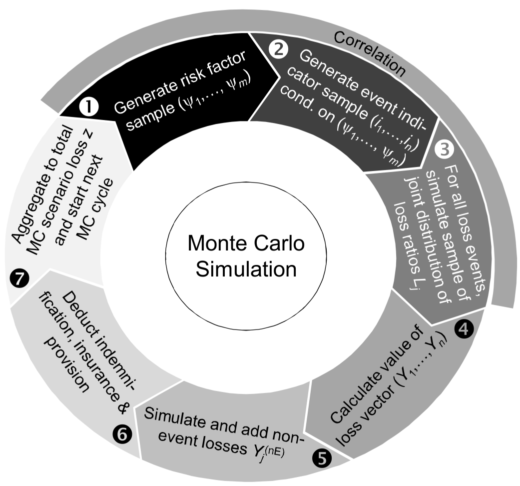

Figure 2: Process flow for generating one MC sample of the aggregated loss distribution.

3.5.2 Risk measures and allocation techniques

In the last 20 years, a well-founded mathematical theory has been developed for measuring and allocating risk capital. The most important cornerstone is the formalization of the properties of a coherent risk measure given in (Artzner et al., 1999). Their axiomatization provides an appropriate framework for the analysis and development of risk measures, e.g. Expected Shortfall in (Rockafellar and Uryasev, 2000) and (Acerbi and Tasche, 2002). General principles for capital allocation can be found in a number of papers, for example in (Kalkbrener, 2005).[1]

The general theory of measuring and allocating risk is independent of specific risk types. In particular, these techniques can be applied in EBOR models, e.g. standard risk measures like Value-at-Risk or Expected Shortfall can be derived from the Monte Carlo samples representing the aggregate loss distribution. More precisely, the formulas

$$\begin{aligned} \mathrm{VaR}(\alpha) &:= \inf\{z \in \mathbb{R} \mid \mathbb{P}(Z \leq z) \geq \alpha\}, \\ \mathrm{ESF}(\alpha) &:= \mathbb{E}(Z \mid Z > \mathrm{VaR}(\alpha)) \end{aligned}$$

can be numerically evaluated if a sample list of the aggregate loss distribution Z has been calculated.[2]

EBOR models allow for allocation of risk capital to individual exposures. This is due to the fact that each sample z of Z can be split into contributions of the n loss events, i.e. $\sum_{j=1}^{n} z_j$, which provides the necessary information for calculating allocation formulas like Expected Shortfall allocation:

$$\mathrm{ESF}_i(\alpha) \quad := \quad \mathbb{E}(Z_i \mid Z > \mathrm{VaR}(\alpha)).$$

Tail-focused allocation techniques like Expected Shortfall based on a high quantile are designed to highlight risk concentrations. If the underlying portfolio is of limited granularity, risk capital is allocated to a small number of portfolio constituents. It depends on risk management strategies whether this feature of tail-focused allocation is desirable or whether alternative techniques are preferred, e.g allocation techniques that give more weight to the body of the underlying distributions. These considerations are particularly relevant for EBOR models since these models are typically designed to quantify specific aspects of operational risk, e.g. litigation risk, which may have rather concentrated risk profiles.

[1] For more recent research on capital allocation techniques we refer to (Centrone and Gianin, 2017) and the papers cited therein.

[2] Note that the formula for $\mathrm{ESF}(\alpha)$ is an approximation of the coherent risk measure Expected Shortfall, which is defined by $(1-\alpha)^{-1} \int_{\alpha}^{1} \mathrm{VaR}(u)\, du$.

4 EBOR model for litigation risk

In this section we illustrate the application of the general EBOR concept to a portfolio of pending litigations.

Over the last decade litigation risk has become a major driver for banks' OR. A recent note by the Boston Consulting Group, see (Grasshoff et al., 2017), shows that banks across the world have paid "cumulative financial penalties of about USD 321 billion assessed since the 2007-2008 financial crisis through the end of 2016". The reflection of this risk under a classical LDA approach, however, suffers from a number of issues we previously described in section 2.1. Summarizing, risk is often underestimated at the beginning, since there is no capital impact until the initial provision is established, and overcapitalized after the loss has materialized. On the other hand, the bank has often an in-depth knowledge about the underlying risk, e.g. the likelihood and amount of a payment, as illustrated by the following examples. Event probabilities p_j can be parameterized from categories of "probability of outflow" into which all (material) litigations are classified under existing accounting rules. The (non-inflated) claimed amount may provide an estimate for the maximum loss, i.e., the exposure E_j. Legal fees are projected in the budgeting process. Finally, potential risk mitigants like eligibility for third party indemnification or insurance or existing provisions are explicitly known for each litigation.

In conclusion, (pending) litigations form a prime example for an exposure-based treatment within the OR framework also following the set of criteria developed in section 2.3. In the next subsections, we will specify an EBOR model for a "litigation portfolio" starting with its description and the definition of a loss event in the context of litigations in section 4.1. We continue with data requirements (section 4.2) and the corresponding model specification (section 4.3). In the final section 4.4 we illustrate the model behavior for a typical litigation over the hypothetical states in its life cycle.

4.1 Basic portfolio definition and model variables

Throughout the section we consider a "portfolio" of n pending litigations. A *loss event* is defined as the occurrence of a payment due to settlement or negative court ruling within the capital horizon of one year. For the j-th litigation it is indicated by the Bernoulli variable I_j. The random quantity to be paid in case of an event, the outflow, is denoted by O_j; it equals the product of deterministic exposure E_j and a stochastic loss-given-event ratio L_j.[3] In addition to those potential event losses, we model non-event losses $Y_j^{(\mathrm{nE})}$ in terms of legal fees denoted by $\mathrm{LegalFee}_j$, $j = 1\ldots,n$. Fees and event losses are subject to risk mitigation through indemnification, insurance or provisions. In this section, we focus on modeling (correlated) event

[3]Note that the total (final) loss for a litigation is simulated in our model, not incremental P&L hits per year like the establishment of (additional) provisions.

losses and independent legal fees for litigations, namely the components of

$$Z_j^{(0)} = \text{LegalFee}_j + Y_j = \text{LegalFee}_j + I_j \cdot L_j \cdot E_j,$$

while the specification of risk mitigants from the relevant data is typically rather straightforward and therefore not covered.

4.2 Data requirements

In the following we outline which type of data is required to parameterize the EBOR model for the litigation portfolio. Data requirements cover case-specific information for each litigation as well as identification of dependencies (*contagion effects*) across the portfolio. In the general case, the required set of data should be readily available since it is likely to be part of existing data processes, like a case classification required for accounting or a specification of outflow estimates in the provisioning process. It should hence be primarily provided by the SMEs in the bank. Whenever it cannot be sourced this way, it would need to be calibrated from historic internal and external recordings, e.g. (Boettrich and Starykh, 2017) or using classical operational loss data. The existence of such a database also serves the purpose to challenge and validate the estimates set by the SME.

4.2.1 Case-specific information

For each pending litigation we require a number of parameter specifications described in detail below. Those complement readily available figures like existing provisions P_j or the information whether a case is eligible for indemnification or insurance (with corresponding parameterizations).

Probability of outflow: The probability of outflow $p_j^{(\text{total})}$ describes the probability that a loss event for the j-th litigation will happen at all (without timing restrictions). As a first classification, we propose to use the categories set under the accounting rules. For instance, the "International Accounting Standard 37", see (International Accounting Standard Board, 1998), requires a classification of pending legal cases with respect to their likelihood, i.e., whether associated losses are 'probable', 'more than remote less than probable' or 'remote'. The IAS does not explicitly specify probabilities next to those categories and an assignment is considered to be a difficult task in general. However, a reasonable choice could be to assume that the probability of outflow is bigger than 50% for probable cases, while its upper bound for remote cases is in a range of 5% to 15%. One can then either set $p_j^{(\text{total})}$ to a common value by category or vary the estimate by case, e.g. by including additional information about the likelihood of a loss event.

Remaining life-time: The remaining life-time $\hat{T}_j$ of the litigation estimates the time until closure of the case. It often depends on the age of the litigation and is required for the annualization of the probability of outflow.

Outflow estimates: We need estimates of the amount of outflow, O_j, for a loss event. The following list provides some examples:

- maximum amount of outflow (e.g. claimed amount),
- expected amount of outflow (e.g. amount used for provisions),
- quantile estimate, i.e., amount H_j that the outflow will not exceed with a likelihood of $h_j \in [0, 1]$.

In setting those estimates a number of factors are considered. Examples are type of claim, the plaintiff, the status of the case, rulings on dispositive motions, prior settlement discussions, other relevant rulings by courts or arbitration panels, relevant factual and legal developments, relevant settlements by others, the schedule for the litigation or arbitration and opposing counsel. Clearly, the estimates are subject to significant judgment, and contemplate a variety of assumptions, variables and known and unknown uncertainties.

The number of required outflow estimates depends on the distributional form of the outflow variable or, equivalently, the loss ratio. As we will show in the next subsection, our model specification requires at least two estimates. Any additional information on the shape of the distribution improves the reflection of risk.

Legal fee estimates: Legal fees are, in general, paid on a regular basis, regardless whether a loss event has occurred or not. They are usually part of the budgeting process. Estimates of a similar kind are available as for the outflow estimates above. Within our model specification we require at least two estimates.

4.2.2 Identification of dependencies between litigations

Usually, litigations can be grouped into clusters in the following sense: Any court ruling for one litigation impacts the likelihood of a payment for other litigations in the same cluster and, on the other hand, does not influence the outcome of litigations outside the cluster, e.g. the same line of arguments might be applicable to litigations in the same cluster but not to the rest. This set-up nicely ties in with the clustering approach described in section 3.1.1. The SME is asked to specify a partition $C_1, \ldots, C_m$ of $\{1, \ldots, n\}$ splitting the portfolio into m litigation clusters. Within each cluster maximum dependence between events is assumed, see section 3.1.1, while litigations in different clusters are supposed to be independent.

The clustering is also used to parameterize the correlation matrix $\Sigma_L \in \mathbb{R}^{n \times n}$ for loss ratio modeling. For each cluster C_i, $i = 1, \ldots, m$, we ask for a correlation parameter ϱ_i such that $\mathrm{Corr}(L_{j_1}, L_{j_2}) = \varrho_i$ for all $j_1 \neq j_2 \in C_i$. In practice, this estimate is expected to be difficult to calibrate from historic observations and should be set by the SMEs.

4.3 Model specification

4.3.1 Correlated litigation event losses

As introduced in section 4.1, litigation event losses are modeled in terms of the event indicators I_j and the amount of outflow O_j. We next introduce the model specification for those model variables.

For any event indicator we need to estimate its expected value, in other words, the event probability p_j. It is derived by annualizing the related (multi-year) probability of outflow $p_j^{(\text{total})}$. We first translate the estimated remaining life-time $\hat{T}_j$ of the litigation into the time-adjustment factor $\mathrm{TAF}_j \in [0, 1]$ and then set the event probability to the product

$$p_j := \mathrm{TAF}_j \cdot p_j^{(\text{total})}, \quad j \in \{1, \dots, n\}. \tag{11}$$

Note that the high values for the probability of outflow buckets constitute a fundamental difference to standard credit risk modeling, where typical default probabilities are significantly lower. This feature should be kept in mind when comparing characteristics of credit portfolio models and EBOR models for litigation risk.

The correlation of different litigation events is captured through the clustering of litigations, see section 4.2.2. Model details are provided in section 3.1.1.

Given a loss event for the j-th litigation, represented by $I_j = 1$, we need to model the uncertainty in the final payment or amount of outflow O_j. In line with the general EBOR model concept, we have defined this random quantity as the product of a deterministic exposure E_j and the stochastic loss ratio L_j. While we have (in most cases) an expert opinion on the expected value as well as the upper and/or lower bound of O_j, we still need to model the uncertainties around these estimates, e.g., the volatility of L_j. In order to specify the distribution functions F_{L_j}, the stochastic loss ratios L_j are fitted to internal and external recordings of loss ratios from litigation events as listed in (Boettrich and Starykh, 2017). We found that log-normal distributions often provide a good fit:

$$L_j \sim \mathrm{Lognormal}(\mu_j^{(\text{LGE})}, \sigma_j^{(\text{LGE})}).$$

This implies that the amount of outflow O_j also follows a log-normal distribution with

$$\mathbb{E}[O_j] = E_j \cdot \mu_j^{(\text{LGE})}, \;\; \sigma(O_j) = E_j \cdot \sigma_j^{(\text{LGE})}. \tag{12}$$

and we calibrate the mean $\mu_j^{(\mathrm{LGE})}$ and volatility $\sigma_j^{(\mathrm{LGE})}$ as well as the exposure E_j to the set of available estimates on O_j.

We give two concrete examples. First, we suppose that there is an expert assessment of the claimed amount and the expected amount of outflow $\mathrm{eO_j}$ (e.g. from provisioning). In this case we set the exposure E_j to the claimed amount and calibrate the volatility $\sigma^{(1)} = \sigma(O_j)$ such that the condition

$$O_j \overset{a.s.}{\leq} E_j$$

is fulfilled for a log-normal distribution with mean $\mathrm{eO_j}$.[4] From these conditions, we finally derive $\mu_j^{(\mathrm{LGE})}$ and $\sigma_j^{(\mathrm{LGE})}$, see equation (12). Note that the exposure could also include some buffer if there is a chance that the claimed amount may be exceeded.

In a variation, we assume that, in addition, the SME estimates that the final payment will not exceed some upper bound $\mathrm{uO_j}$ with a probability of $q(\mathrm{uO}_j) \in (0, 1)$:

$$\mathbb{P}[O_j \leq \mathrm{uO_j}] = \mathrm{q(uO_j)}. \tag{13}$$

The quantile $q(\mathrm{uO}_j)$ might vary with the litigation type or the expected remaining life time of the litigation. For instance, loss estimation for regulatory matters are sometimes subject to a comparatively higher uncertainty. We next calibrate a second volatility $\sigma^{(2)} = \sigma(O_j)$ such that the conditions

$$\mathbb{E}[O_j] = \mathrm{eO_j}, \;\; \mathbb{P}[\mathrm{O_j} \leq \mathrm{uO_j}] = \mathrm{q(uO_j)}$$

are fulfilled. We then need to decide which volatility estimate, $\sigma^{(1)}$ or $\sigma^{(2)}$, is more adequate to model the uncertainty in outflow estimates.

Finally, the joint simulation of the ratios is based on a Gaussian copula

$$C_L(x_1, \ldots, x_n) = C_{\Sigma_L}^{(\mathrm{Ga})}(F_{L_1}(x_1), \ldots, F_{L_n}(x_n))$$

with correlation matrix $\Sigma_L \in \mathbb{R}^{n \times n}$ specified in section 4.2.2.

4.3.2 Non-event losses: Legal fees

In our model, non-event losses $Y_j^{(\mathrm{nE})}$ are represented by legal fees. Unlike litigation payments, legal fees are supposed to occur on an pre-determined regular basis and independently from each other. Loss distributions arising from legal fees are generally supposed to be less heavier in the tail than those reflecting the uncertainty in final litigation payments for which we utilized a log-normal distribution. Therefore, we assume that realizations of next year's legal fees follow a normal distribution:

$$\mathrm{LegalFee}_j \sim \mathcal{N}\left(\mu_j^{(\mathrm{LegalFee})}, \sigma_j^{(\mathrm{LegalFee})}\right), j \in \{1, \ldots, n\}.$$

Similar to the parameter calibration for loss ratios L_j, we require at least two loss estimates for legal fees. Their expected value and volatility are then derived from those estimates. Additionally, we floor all realizations of fee losses at zero.

[4]Practically, we impose the conditions $\mathbb{E}[O_j] = \mathrm{eO_j}$ and $\mathbb{P}[O_j \leq E_j] = 1 - \varepsilon$ for some small $\varepsilon > 0$.

4.4 Model illustration

In order to demonstrate its favorable properties, we illustrate the output of a (stand-alone) EBOR model for a hypothetical litigation over different states in its life cycle. Note that the life cycle parameters have been artificially chosen to emphasize certain model sensitivities and capital effects and have not been derived from real litigations.

We divide the life cycle into five different phases from initial filing (phase 1) to final payment and closure of the matter (phase 5). Since these phases typically do not coincide in length we ignore, for the sake of illustration, the annualization of event probabilities with respect to the estimated remaining life-time of the litigation. We also ignore legal fees as well as indemnification and insurance, focusing only on potential event losses plus the effect of provisioning. The loss variable is hence given by

$$Z = Y - I \cdot P = I \cdot (L \cdot E - P) = I \cdot (O - P)\,.$$

The estimated exposure of the hypothetical litigation is 1bn. Since little is known about the case at the time of the initial filing, it is classified as "remote" and the event probability is estimated as 5%. The expected loss ratio is calibrated to historic data for cases of similar kind. Let us assume a value of 20%. Correspondingly, the expected amount of outflow equals 200mn. In phase 2, e.g. after one month, classification changes. The case has been reviewed and is now considered "more than remote less than probable" with the event probability increasing to 30%. No specific provision is made but the estimated expected loss is disclosed (in an aggregated way) as so-called contingent liability, see (International Accounting Standard Board, 1998). It is assumed by the SME that the amount of outflow will be less than 400mn with a likelihood of 90%. Afterwards, in phase 3, the estimated range of outflow narrows as more detailed information becomes available. The expected amount of outflow increases from 200mn to 300mn and the quantile estimate for 400mn changes to 95%. In the fourth phase, the SME classifies the case as "probable" and a provision P is established which equals the current expected amount of outflow of 300mn. The event probability increases to 75%. Furthermore, the likelihood that the amount of outflow will be above 400mn is now considered to be 1%. We finally assume the event is settled for an amount of 350mn in phase 5.

We now illustrate how the different phases of the life cycle of a litigation are reflected in an EBOR model. For the entire life of the litigation we set its exposure E to 1bn. The loss ratio is modeled using the technique presented in section 4.3.1. Figure 3 shows the distribution function of the loss ratio over the different phases. One observes that the estimate of the final payment stabilizes over time. For instance, the volatility is highest (18%) in phase 2, due to a comparatively high uncertainty of estimates, and lowers to 3.8% in phase 4 when the case approaches closure.

For the first three phases the distribution function of the loss variable Z resembles the one of the loss ratio. In phase 4, however, we are allowed to deduct the provision

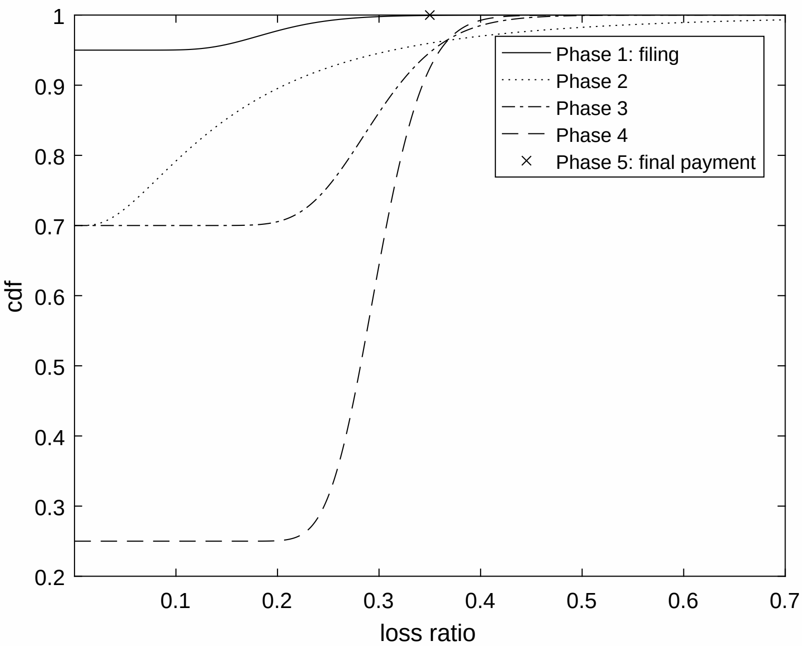

Figure 3: Distribution function (cdf) of the loss ratio over the life cycle of the hypothetical litigation. In the end the case is settled for a rate of 35%.

of 300mn. This corresponds to a shift to the left in the distribution function. Table 1 lists the different key risk measures of the model over the different phases and compares them to actual P&L hits. In particular, the Value-at-Risk (VaR) plus accumulated P&L (in absolute terms) converges to the final settlement amount of 350mn. Note that, once settled in phase (5), the litigation does no longer impact the risk calculated for any remaining litigation.

These characteristics are different to the treatment under LDA, where historic cases determine the frequency and severity variables specified for litigation risk. Additionally, the litigation would have been ignored under traditional LDA until phase (4), i.e. the establishment of a provision, leading to undercapitalization at the beginning of the litigation life cycle and overcapitalization after the loss materializes. In fact, it is a counterintuitive phenomenon that provisioning, which reduces risk capital in a realistic model, leads to an increase in LDA models. This features illustrate the LDA shortcomings discussed in section 2.1.

Phases	1	2	3	4	5
Input model parameters:					
p	5%	30%	30%	75%	100%
E (in mn)	1000	1000	1000	1000	1000
eO (in mn)	200	200	300	300	350
uO (in mn) / q(uO)	—	400 / 90%	400 / 95%	400 / 99%	—
P (in mn)	0	0	0	300	0
Derived model parameters:					
$\mu(L)$	20%	20%	30%	30%	35%
$\sigma(L)$	5.3%	18.2%	5.6%	3.8%	0%
Model output (in mn):					
accumulated P&L	0	0	0	-300	-350
EL	10	60	90	0	0
VaR (99.5%)	269	771	437	108	0

Table 1: Model output (Expected Loss and Value-at-Risk) vs. accumulated P&L impact due to provisioning (phase 4) and final payment (phase 5).

5 Integration of LDA and EBOR

5.1 Integrated simulation of OR events under LDA and EBOR

A sound OR capital calculation has to be conducted in a fully integrated and diversified way. As a consequence, the integration of LDA and EBOR models becomes an important component of OR quantification if both approaches are applied, e.g. EBOR models are used for predictable risk types whereas LDA models cover risks that are well reflected through historical events.

In the LDA model the dependence structure is specified for the units of measure, which we refer to as cells, e.g. specified on BL/ET level. The precise definition of the LDA dependence structure has a significant impact on the integration strategy. In this paper, we focus on the LDA model specified in (1) and assume that the frequency variables N_j, $j = 1, \ldots, n$, are correlated through a copula whereas the severity variables S_{jk} are independent, see (Aue and Kalkbrener, 2006). In this setup, we integrate the EBOR model by specifying the dependence of the EBOR frequency and LDA frequency variables through an additional dimension of the copula, i.e., the dimension of the copula is increased to $n+1$. In other words, the EBOR model is considered as an additional cell, analogously to the BL/ET combinations in a classical LDA model.[5]

[5]Other frequently used LDA models are based on dependence structures applied to aggregated cell loss distributions, which combine frequency and severity information. The integration of an

Please note that any integration concept would also trigger changes of the LDA model or its input data, e.g. to avoid double counting of loss potential. Here, we assume that this clear separation of LDA or EBOR events has been accomplished beforehand leaving us with the task of specifying the integration model. For the sake of simplicity, risk mitigation effects are also ignored.

In our set-up we assume that we have r cells remaining under the LDA approach ("residual LDA cells") and a $(r+1)$-th cell treated under the EBOR approach ("EBOR cell"). The restriction to one EBOR cell is for illustration purposes only. The concept naturally prolongs to an arbitrary number of EBOR cells. With respect to frequencies, we suppose that the EBOR cell is exposed to a maximum of n events. The event frequencies of all cells are linked via a copula

$$C^{(\mathrm{Freq})}_{\Sigma^{(\mathrm{Freq})}}(F^{(\mathrm{Freq})}_1(x_1), \ldots, F^{(\mathrm{Freq})}_{r+1}(x_{r+1}))$$

with $F^{(\mathrm{Freq})}_l$ denoting the event frequency distribution of the l-th cell and $\Sigma^{(\mathrm{Freq})}$ being the correlation matrix.

For each scenario ℓ of our Monte Carlo simulation, we obtain a sample

$$(u_1(\ell), \ldots, u_r(\ell), u_{r+1}(\ell))$$

of correlated uniform random variables. They are cell-wise translated into a number of loss events by applying the inverse of the cell frequency distribution function:

$$n_l(\ell) := \left(F^{(\mathrm{Freq})}_l\right)^{-1}(u_l(\ell)), \quad l = 1, \ldots, r+1.$$

Each loss event has a certain severity, i.e. $S_l(1), \ldots, S_l(n_l(\ell))$ for the l-th cell, whose determination can now differ. For the first r cells treated under LDA, losses will remain to be sampled from a dedicated severity distribution:

$$S_l(\ell) = \sum_{j=1}^{n_l(\ell)} S_l(j).$$

For the $r+1$-th cell we are combining losses from EBOR events:

$$S_{r+1}(\ell) = Z(\ell) = \sum_{j=1}^{n} Y^{(\mathrm{nE})}_j(\ell) + I_j(\ell) \cdot L_j(\ell) \cdot E_j.$$

Note that the realizations of $I_1(\ell), \ldots, I_n(\ell)$ amount to a total of $n_{r+1}(\ell)$.

Cell losses are finally aggregated yielding the portfolio scenario loss

$$S(\ell) := \sum_{l=1}^{r+1} S_l(\ell)$$

EBOR model into such a framework would be simpler since the most complex task for integration – the recursive simulation of EBOR events starting from a total event frequency– can be omitted.

The realizations $S(\ell)$ form the basis to estimate the loss distribution from which we can read off risk measures like Value-at-Risk.

Most of the procedures above are standard and straight-forward to apply. The only missing piece is the translation of the total number of EBOR events, $n_{r+1}(\ell)$, into realizations of $I_1(\ell), \ldots, I_n(\ell)$. This will be shown in next section.

5.2 Recursive simulation of EBOR events

We describe a simple algorithm for a recursive simulation of EBOR events.

The simulation of EBOR loss events is based on the cluster densities $f_1, \ldots, f_m$ and cumulative cluster densities $g_1, \ldots, g_m$ defined in section 3.1.1. Formula (7) explicitly specifies the densities $f_1, \ldots, f_m$, whereas the recursion formula (8) can be easily used to calculate the cumulative densities $g_1, \ldots, g_m$. Note that the frequency distribution function $F_{r+1}^{(\mathrm{Freq})}$ is specified by the cumulative density g_m via

$$F_{r+1}^{(\mathrm{Freq})}(k) = \sum_{j=0}^{k} g_m(k), \quad k = 0, \ldots, n.$$

As shown in the previous subsection, $F_{r+1}^{(\mathrm{Freq})}$ is used to perform a joint simulation of EBOR and LDA frequency distributions, which are linked through a copula function. The output is a total number of EBOR events, $n_{r+1}(\ell)$, for a specific scenario. It remains to simulate the corresponding EBOR events, i.e. determine a joint state for the event indicator variables $(I_1, \ldots, I_n)$ such that

$$I_P(\ell) = \sum_{j=1}^{n} I_j(\ell) = n_{r+1}(\ell).$$

The simulation algorithm is based on an iterative application of recursion formula (8) starting with the calculation of events in cluster C_m, then proceeding with C_{m-1} and so on.

We first set $k_m := n_{r+1}(\ell)$. Conditional on $I_P = k_m$ the frequency variable I_{C_m} has the form

$$\mathbb{P}\left(I_{C_m} = j \mid I_P = k_m\right) = \left(g_{m-1}(k_m - j)\, f_m(j)\right)/g_m(k_m),$$

where j is an element of $\{0, \ldots, |C_m|\}$. We sample this random variable and obtain $l_m \in \{0, \ldots, |C_m|\}$, which denotes the number of losses in cluster C_m. According to the dependence structure in C_m losses have occurred for the l_m cluster elements with highest event probabilities.

The number of losses in the remaining $m-1$ clusters is $k_{m-1} := k_m - l_m$. We apply recursion formula (8) to cluster C_{m-1}, i.e. we sample $I_{C_{m-1}}$ conditional on $I_P = k_m$ and $I_{C_m} = l_m$, and obtain the loss events in cluster C_{m-1}. After repeating the procedure for the remaining clusters the full set of EBOR loss events has been specified.

6 Conclusion

In the preceding sections we have introduced the general framework for an exposure-based approach in the context of OR quantification. We have described the general methodological framework highlighting the benefits of the new approach, in particular with respect to the commonly used LDA. We have detailed the application of EBOR to a portfolio of pending litigations. This risk type is particularly well-suited for an exposure-based approach due to better usage of existing information and more plausible model behavior over the litigation life cycle. Lastly, we have discussed a strategy how to integrate EBOR and LDA models building hybrid frameworks which facilitates the migration of OR subtypes from a classical to an exposure-based treatment.

As major advantage of EBOR we would like to emphasize the wide scope of applicability of EBOR beyond capital calculation and its potential to evolve into an important OR management tool - accepted by risk managers, business experts as well as quants. The aim of this paper is to contribute to a conceptual / terminological basis for future development across industry helping to establish a common language. We would like to encourage further advances in the direction of EBOR modeling and point out the importance of joint industry efforts (e.g. through industry forums) to prove the successful applicability of EBOR and to convince competent authorities that EBOR should be considered as valuable instrument for future OR measurement and management.

Declaration of interest

The authors report no conflicts of interest. The authors alone are responsible for the content and writing of the paper. The views and opinions expressed in this paper are those of the authors in a personal capacity and do not necessarily reflect those of Deutsche Bank. Examples of analysis performed within this paper are only examples and do not concern actual data, processes or procedures. Consequent insights made within the analysis are not reflective of positions at Deutsche Bank. Any litigation data or examples provided are purely illustrative and completely unrelated to any real matters, events or processes.

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